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PQ - Sum Divisor Cordial graphs

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Abstract:
Graph labeling is an assignment of integers to the vertices or edges or both depending on certain conditions. A graph G with p vertices and q edges is said to admit PQ- sum divisor cordial labeling if the labeling h from V(G) to $\{1, 2, \dots, p\}$ induces a mapping $h^*: E(G) \to \{0,1\}$ as $h^*(xy) = \begin{cases} 1 & if\ 2 \mid (P_{xy} + Q_{xy}) \\ 0 & otherwise \end{cases}$ with the condition that $|e_{h^*}(0) - e_{h^*}(1)| \le 1$, where $e_{h^*}(k)$ is the number of edges labeled with k. A graph which admits a PQ- sum divisor cordial labeling is called a PQ- sum divisor cordial graph. In this paper, we prove that the path P_n, cycle C_n, star graph $K_{1,n}$, bistar graph $B_{m,n}$, the subdivision graph of the star and bistar graphs $S(K_{1,n})$ and $S(B_{m,n})$, the splitting graph of the star graph $S'(K_{1,n})$, the fan graph $F_{1,n}$, the vertex switching of the path and the cycle, the graphs $P_n^{\ 2}$ and $P_n \odot K_1$ are PQ- sum divisor cordial graphs.

Keywords: labeling, cordial labeling, sum divisor cordial labeling.

1. INTRODUCTION

Labeling of a graph is an immense and vast area of research in the field of graph theory. If the vertices or edges or both of a graph are assigned values subject to certain conditions, then it is known as graph labeling. Cahit proposed the notion of cordial labeling in 1987 as a weaker version of graceful and harmonious labeling [1]. Let f be a function from the vertex set of G to $\{0, 1\}$ and for each edge xy assign the label $|f(x) - f(y)|$. The function f is called a cordial labeling of G if the number of vertices labeled with 0 and the number of vertices labeled with 1 differ at most by 1 and number of edges labeled with 0 and the number of edges labeled with 1 differ at most by 1. The notion of sum divisor cordial labeling was introduced by A. Lourdusamy and F. Patrick[4]. Let f be a bijection from the vertex set of G to $\{1, 2, \dots, |V(G)|\}$ and for each edge xy assign the label 1 if 2 divides $f(x) + f(y)$ and the label 0 otherwise. The function f is called a sum divisor cordial labeling of G if the number of edges labeled with 0 and the number of edges labeled with 1 differ at most by 1. Motivated by this we introduced PQ- sum divisor cordial labeling of graphs. In this section we provide a summary of definitions and notations required for our investigation.

Definition 1.1. The subdivision graph $S(G)$ is obtained from a graph G by subdividing each edge of G with a vertex.

Definition 1.2. For a graph G the splitting graph $S'(G)$ is obtained by adding a new vertex x' corresponding to every vertex x of G such that $N(x) = N(x')$, where $N(x)$ is the set of all vertices adjacent to x in G.

Definition 1.3. The corona product $G_1 \odot G_2$ of two graphs $G_1(p_1, q_1)$ and $G_2(p_2, q_2)$ is defined as the graph obtained by taking one copy of G_1 and p_1 copies of G_2 and joining i^{th} vertex of G_1 with an edge to every vertex in the i^{th} copy of G_2.

Definition 1.4. The fan graph $F_{m,n}$ is defined as the join $\overline{K_m} + P_n$, where $\overline{K_m}$ is the trivial graph on m vertices and P_n is the path graph on n vertices.

Definition 1.5. The square graph G^2 of a graph G is obtained from G by adding new edges between every two vertices having distance two in G.

Definition 1.6. The vertex switching G_v of a graph G is the graph obtained by removing all the edges incident with the vertex v of G and joining the vertex v to every vertex which is not adjacent to v by an edge.

Notation 1.7. Let h be a vertex labeling and $xy \in E(G)$. We denote $P_{xy} = h(x)h(y)$ and

$$Q_{xy} = \begin{cases} \left\lfloor \dfrac{h(x)}{h(y)} \right\rfloor & if \ h(x) > h(y) \\ \left\lfloor \dfrac{h(y)}{h(x)} \right\rfloor & if \ h(y) > h(x) \end{cases} \quad \text{for all } xy \in E(G).$$

Notation 1.8. Let us denote $e_{h^*}(k)$ = the number of edges labeled with k.

Definition 1.9. Let G be a simple graph and $h : V(G) \to \{1, 2, \ldots, |V(G)|\}$ be a bijection. For each edge xy assign $h^*(xy) = \begin{cases} 1 & if \ 2 \mid (P_{xy} + Q_{xy}) \\ 0 & otherwise \end{cases}$. The labeling h is called a PQ- sum divisor cordial labeling if $|e_{h^*}(0) - e_{h^*}(1)| \le 1$. A graph which admits a PQ- sum divisor cordial labeling is called a PQ- sum divisor cordial graph.

2. MAIN RESULTS

Theorem 2.1. The path graph P_n is a PQ- sum divisor cordial graph.

Proof. Let G be the path graph P_n. Let $x_i (1 \le i \le n)$ be the vertices of G.

Define $h: V(G) \to \{1, 2, \dots, n\}$ by $h(x_1) = 1, h(x_2) = 2$ and

$$h(x_i) = \begin{cases} 2i-3 & if\ 3 \le i \le \left\lceil \frac{n}{2} \right\rceil + 1 \\ 2(n-i)+4 & if\ \left\lceil \frac{n}{2} \right\rceil + 2 \le i \le n \end{cases}$$

Then the labeling h will induce the map $h^*: E(G) \to \{0, 1\}$. Here, the number of edges labeled with 0 and 1 are $e_{h^*}(0) = e_{h^*}(1) = \frac{n-1}{2}$ if n is odd and $e_{h^*}(0) = \left\lceil \frac{n-1}{2} \right\rceil$, $e_{h^*}(1) = \left\lfloor \frac{n-1}{2} \right\rfloor$ if n is even. Thus $| e_{h^*}(0) - e_{h^*}(1) | \le 1$ and hence P_n is a PQ- sum divisor cordial graph.

Theorem 2.2. The complete graph K_n is not a PQ- sum divisor cordial graph for all n.

Proof. Let G be the complete graph K_n. Let $x_i (1 \le i \le n)$ be the vertices of G. Label the vertices of G in any order.

Let A = {1, 2, 3, 7}. Then we have $e_{h^*}(1) > \left\lceil \frac{n(n-1)}{4} \right\rceil$ for all $n \le 17$ *and* $n \notin A$. Also, $e_{h^*}(1) < \left\lfloor \frac{n(n-1)}{4} \right\rfloor$ for all $n \ge 18$. Thus, $| e_{h^*}(0) - e_{h^*}(1) | > 1$ for all $n \notin A$. Hence, K_n is not a PQ- sum divisor cordial graph for all n.

Theorem 2.3. The cycle graph C_n is a PQ- sum divisor cordial graph.

Proof. Let G be the cycle graph C_n. Let $x_i (1 \le i \le n)$ be the vertices of G. Define $h: V(G) \to \{1, 2, \dots, n\}$ as follows:

Case(i): n is odd

Label the vertex x_1 by 1, $x_{\left\lceil \frac{n}{2} \right\rceil}$ by n, $x_{\left\lceil \frac{n}{2} \right\rceil + 1}$ by n − 1 and the remaining vertices by

$$h(x_i) = \begin{cases} 2i-2 & if\ 2 \le i \le \left\lfloor \frac{n}{2} \right\rfloor \\ 2(n-i)+3 & if\ \left\lceil \frac{n}{2} \right\rceil + 2 \le i \le n \end{cases}$$

Then the labeling h will induce the map $h^*: E(G) \to \{0, 1\}$. Also, we get $e_{h^*}(1) = \left\lceil \frac{n}{2} \right\rceil$ *and* $e_{h^*}(0) = \left\lfloor \frac{n}{2} \right\rfloor$.

Case(ii): n is even

In this case, label the vertex x_1 by 1, $x_{\frac{n}{2}+1}$ by $n-1$, $x_{\frac{n}{2}+2}$ by n and the remaining vertices by $h(x_i)=\begin{cases}2i-2 & if\ 2\le i\le \dfrac{n}{2}\\ 2(n-i)+3 & if\ \dfrac{n}{2}+3\le i\le n\end{cases}$

Then the number of edges labeled with 0 and 1 are $e_{h^*}(0)=e_{h^*}(1)=\dfrac{n}{2}$.

Thus in each case, we have $|e_{h^*}(0)-e_{h^*}(1)|\le 1$. Hence, C_n is a PQ- sum divisor cordial graph.

Theorem 2.4. The graph $P_n^{\ 2}$ is a PQ- sum divisor cordial graph.

Proof. Let $G=P_n^{\ 2}$. Let $x_i(1\le i\le n)$ be the vertices of G. Then $E(G)=\{x_ix_{i+1}:1\le i\le n-1\}\bigcup\{x_ix_{i+2}:1\le i\le n-2\}$. Define $h:V(G)\to\{1,2,\dots,n\}$ as follows:

Case(i): n is odd

$$h(x_i)=\begin{cases}2i-1 & if\ 1\le i\le \left\lfloor \dfrac{n}{2}\right\rfloor\\ 2(n-i)+2 & if\ \left\lceil \dfrac{n}{2}\right\rceil+2\le i\le n\end{cases},\quad h\left(x_{\left\lceil \frac{n}{2}\right\rceil}\right)=n-1\ and\ h\left(x_{\left\lceil \frac{n}{2}\right\rceil+1}\right)=n.$$

Here, the labeling h will induce the map $h^*:E(G)\to\{0,1\}$. Also, we get $e_{h^*}(1)=n-1\ and\ e_{h^*}(0)=n-2$.

Case(ii): n is even

$$h(x_i)=\begin{cases}2i-1 & if\ 1\le i\le \dfrac{n}{2}\\ 2(n-i)+2 & if\ \dfrac{n}{2}+1\le i\le n\end{cases}$$

In this case, we have $e_{h^*}(1)=n-1\ and\ e_{h^*}(0)=n-2$.

Thus in each case, we have $|e_{h^*}(0)-e_{h^*}(1)|\le 1$. Hence, $P_n^{\ 2}$ is a PQ- sum divisor cordial graph.

Theorem 2.5. The star graph $K_{1,n}$ is a PQ- sum divisor cordial graph.

Proof. Let G be the star graph $K_{1,n}$. Let $x, x_i(1\le i\le n)$ be the vertices of G. Define $h:V(G)\to\{1,2,\dots,n+1\}$ as follows:

Case(i): $n\equiv 0(\text{mod } 4)$

Label the vertex x by 4 and $x_i (1 \le i \le n)$ by 1, 2, 3, 5, 6, ... , n + 1 in any order. Then the number of edges labeled with 0 and 1 are $e_{h^*}(0) = e_{h^*}(1) = \frac{n}{2}$.

Case(ii): $n \equiv 1,2,3 \pmod 4$

Label the vertex x by 2 and $x_i (1 \le i \le n)$ by 1, 3, 4, ..., n + 1 in any order. Then the number of edges labeled with 0 and 1 are $e_{h^*}(0) = \left\lfloor \frac{n}{2} \right\rfloor$ *and* $e_{h^*}(1) = \left\lceil \frac{n}{2} \right\rceil$.

In both cases, we have $| e_{h^*}(0) - e_{h^*}(1) | \le 1$. Hence, $K_{1,n}$ is a PQ- sum divisor cordial graph.

Theorem 2.6. The fan graph $F_{1,n}$ is a PQ- sum divisor cordial graph.

Proof. Let G be the fan graph $F_{1,n}$. Let $x, x_i (1 \le i \le n)$ be the vertices of G. Then $E(G) = \{xx_i : 1 \le i \le n\} \cup \{x_i x_{i+1} : 1 \le i \le n-1\}$. Define $h : V(G) \to \{1, 2, \ldots, n+1\}$ by $h(x) = 1$ and $h(x_i) = i + 1$. Then the labeling h will induce the map $h^* : E(G) \to \{0, 1\}$ and we get $e_{h^*}(1) = n, e_{h^*}(0) = n-1$.

Here, $| e_{h^*}(0) - e_{h^*}(1) | \le 1$. Hence, $F_{1,n}$ is a PQ- sum divisor cordial graph.

Theorem 2.7. The comb graph $P_n \odot K_1$ is a PQ- sum divisor cordial graph.

Proof. Let G be the comb graph $P_n \odot K_1$. Let $x_i, y_i (1 \le i \le n)$ be the vertices of G. Then $E(G) = \{x_i y_i : 1 \le i \le n\} \cup \{x_i x_{i+1} : 1 \le i \le n-1\}$.

Define $h : V(G) \to \{1, 2, \ldots, 2n\}$ by $h(x_i) = 2i - 1$ and $h(y_i) = 2i$. Then the induced map $h^* : E(G) \to \{0, 1\}$ satisfies $e_{h^*}(1) = n$ *and* $e_{h^*}(0) = n-1$. Here, $| e_{h^*}(0) - e_{h^*}(1) | \le 1$.

Hence, $P_n \odot K_1$ is a PQ- sum divisor cordial graph.

Theorem 2.8. The bistar graph $B_{m,n}$ is a PQ- sum divisor cordial graph.

Proof. Let G be the bistargraph $B_{m,n}$. Let $x, y, x_i (1 \le i \le m), y_j (1 \le j \le n)$ be the vertices of G. Without loss in generality we may assume that $m \ge n$. Define $h : V(G) \to \{1, 2, \ldots, m+n+2\}$ as follows:

Case(i): Both m and n are odd

$h(x_1) = 1, h(x_2) = 3, h(x) = 2, h(y) = 4, h(x_i) = n + 2 + i \quad for\ n+2 \le i \le m$,

$$h(x_i) = \begin{cases} 4\left\lceil \frac{i}{2} \right\rceil - 1 & if\ i\ is\ odd \\ 2i & if\ i\ is\ even \end{cases} \text{ for } 3 \le i \le n+1 \text{ and } h(y_j) = \begin{cases} 4\left\lceil \frac{j}{2} \right\rceil + 1 & if\ j\ is\ odd \\ 2j+2 & if\ j\ is\ even \end{cases} \text{ for } 1 \le j \le n$$

Then the labeling h will induce the map $h^*: E(G) \to \{0, 1\}$. Also the number of edges labeled with 0 and 1 are $e_{h^*}(0) = \left\lceil \frac{m+n+1}{2} \right\rceil$ *and* $e_{h^*}(1) = \left\lfloor \frac{m+n+1}{2} \right\rfloor$.

Case(ii): Both m and n are even

In this case, label the vertices of G as in case(i). Then the number of edges labeled with 0 and 1 are

$$e_{h^*}(0) = \begin{cases} \left\lceil \frac{m+n+1}{2} \right\rceil & if \ n \equiv 2 (\text{mod } 4) \\ \left\lfloor \frac{m+n+1}{2} \right\rfloor & if \ n \equiv 0 (\text{mod } 4) \end{cases}, \quad e_{h^*}(1) = \begin{cases} \left\lceil \frac{m+n+1}{2} \right\rceil & if \ n \equiv 0 (\text{mod } 4) \\ \left\lfloor \frac{m+n+1}{2} \right\rfloor & if \ n \equiv 2 (\text{mod } 4) \end{cases}$$

Case(iii): m is odd and n is even

If $m \equiv 1(\text{mod } 4)$, label the vertices of G as in case(i).

If $m \equiv 3(\text{mod } 4)$ and $n \equiv 2(\text{mod } 4)$, label the vertices of G by

$$h(x_1) = 1, h(y_1) = 3, h(x) = 2, h(y) = 4, h(x_i) = n+2+i \quad for \ n+2 \le i \le m$$

$$h(x_i) = \begin{cases} 2i & if \ i \ is \ odd \\ 2i+1 & if \ i \ is \ even \end{cases} \text{for } 2 \le i \le n \ \text{ and } \ h(y_j) = \begin{cases} 2j+2 & if \ j \ is \ odd \\ 2j+3 & if \ j \ is \ even \end{cases} \quad \text{for } 2 \le j \le n$$

If $m \equiv 3(\text{mod } 4)$ and $n \equiv 0(\text{mod } 4)$, label the vertices of G by

$$h(x_1) = 3, h(y_1) = 1, h(x) = 2, h(y) = 4, h(x_i) = n+2+i \quad for \ n \le i \le m$$

$$h(x_i) = \begin{cases} 2i+2 & if \ i \ is \ odd \\ 2i+3 & if \ i \ is \ even \end{cases} \text{for } 2 \le i \le n-1 \text{ and } h(y_j) = \begin{cases} 2j & if \ j \ is \ odd \\ 2j+1 & if \ j \ is \ even \end{cases} \quad \text{for } 2 \le j \le n$$

Here, we observe that $e_{h^*}(0) = e_{h^*}(1) = \frac{m+n+1}{2}$.

Case(iv): m is even and n is odd

If $[m \equiv 0(\text{mod } 4) \ and \ n \equiv 3(\text{mod } 4)] \ or \ [m \equiv 2(\text{mod } 4) \ and \ n \equiv 1(\text{mod } 4)]$, label the vertices of G as in case(i). If $[(m \equiv 0(\text{mod } 4)$ and $n \equiv 1(\text{mod } 4)]$ or $[m \equiv 2(\text{mod } 4)$ and $n \equiv 3(\text{mod } 4)]$, label the vertices of G as in the case $m \equiv 3(\text{mod } 4)$ and $n \equiv 2(\text{mod } 4)$ of case(iii).

Then the number of vertices and edges labeled with 0 and 1 are $e_{h^*}(0) = e_{h^*}(1) = \frac{m+n+1}{2}$.

In each cases, $|e_{h^*}(0) - e_{h^*}(1)| \le 1$. Hence, $B_{m,n}$ is a PQ- sum divisor cordial graph.

Theorem 2.9. The graph $S(K_{1,n})$ is a PQ- sum divisor cordial graph.

Proof. Let G be the subdivision graph of the star graph $K_{1,n}$. Let $x, x_i (1 \le i \le n)$ be the vertices of $K_{1,n}$ and let $x_i' \ (1 \le i \le n)$ be the vertices which subdivides the edges $xx_i (1 \le i \le n)$. Define $h: V(G) \to \{1, 2, \ldots, 2n+1\}$ by $h(x) = 1, h(x_i) = 2i+1 \ and \ h(x_i') = 2i$. Then h will induce the map $h^*: E(G) \to \{0,1\}$ and we get $e_{h^*}(0) = e_{h^*}(1) = n.$

Here, $|e_{h^*}(0)-e_{h^*}(1)|\le 1$. Hence, $S(K_{1,n})$ is a PQ- sum divisor cordial graph.

Theorem 2.10. The graph $S(B_{m,n})$ is a PQ- sum divisor cordial graph.

Proof. Let G be the subdivision graph of the bistar graph $B_{m,n}$. Let $x, y, x_i(1\le i\le m)$ and $y_j(1\le j\le n)$ be the vertices of $B_{m,n}$. Let z be the vertex which subdivides the edge xy and let $x_i'(1\le i\le m),\ y_j'(1\le j\le n)$ be the vertices which subdivides the edges $xx_i(1\le i\le m)$ *and* $yy_j(1\le j\le n)$ respectively. Define $h:V(G)\to\{1, 2, \dots, 2(m+n)+3\}$ as follows:

Case(i): n is even

$h(x)=1, h(y)=2, h(z)=3, h(x_i)=2i+3, h(x_i')=2(i+1)$ for $1\le i\le m$,

$$h(y_j)=\begin{cases}2(m+j)+2 & if\ \ j\ is\ odd\\ 2(m+j)+3 & if\ \ j\ is\ even\end{cases}\quad and\quad h(y_j)=\begin{cases}2(m+j)+4 & if\ \ j\ is\ odd\\ 2(m+j)+1 & if\ \ j\ is\ even\end{cases}\quad \text{for} 1\le j\le n$$

Case(ii): n is odd

Subcase(i): m is odd

In this case, label the vertices $x, y, z, x_i, x_i'(1\le i\le m-2), y_j, y_j'(1\le j\le n-2)$ as in case(i). Also label $h(y_{n-1})=2(m+n)+3, h(y_n)=2(m+n+1), h(y_{n-1}')=2(m+n)-1$ and $h(y_n')=2(m+n)+1$.

Subcase(i): m is even

In this case, label the vertices $x, y, z, x_i, x_i'(1\le i\le m-1), y_j, y_j'(1\le j\le n-1)$ as in case(i). Also label $h(y_n)=2(m+n)+3$ *and* $h(y_n')=2(m+n+1)$.

In each case, the number of edges labeled with 0 and 1 are $e_{h^*}(0)=e_{h^*}(1)=m+n+1$.

Here, $|e_{h^*}(0)-e_{h^*}(1)|\le 1$. Hence, $S(B_{m,n})$ is a PQ − sum divisor cordial graph.

Theorem 2.11. The graph $S'(K_{1,n})$ is a PQ- sum divisor cordial graph.

Proof. Let G be the splitting graph of the star graph $K_{1,n}$. Let $x, x_i(1\le i\le n)$ be the vertices of $K_{1,n}$ and let $x', x_i'\ (1\le i\le n)$ be the added vertices corresponding to $x, x_i(1\le i\le n)$ to form G. Define $h:V(G)\to\{1, 2, \dots, n\}$ as follows:

Case(i): $n\equiv 0(\text{mod } 4)$

$h(x_1)=1, h(x_2)=3, h(x)=2, h(x')=4,$

$$h(x_i)=\begin{cases}4\left\lceil\frac{i}{2}\right\rceil-1 & if\ \ i\ is\ odd\\ 2i & if\ \ i\ is\ even\end{cases}\quad \text{for } 3\le i\le n\quad \text{and}\quad h(x_i')=\begin{cases}4\left\lceil\frac{i}{2}\right\rceil+1 & if\ \ i\ is\ odd\\ 2i+2 & if\ \ i\ is\ even\end{cases}\quad \text{for } 1\le i\le n$$

Here, the labeling h will induce the map $h^*: E(G) \to \{0, 1\}$. Also the number of edges labeled with 0 and 1 are $e_{h^*}(0) = e_{h^*}(1) = \frac{3n}{2}$.

Case(ii): $n \equiv 1,2,3 (\text{mod } 4)$

$h(x_1) = 1,\ h(x_1') = 3,\ h(x) = 2,\ h(x') = 4,$

$h(x_i) = \begin{cases} 2i+2 & if\ i\ is\ odd \\ 2i+3 & if\ i\ is\ even \end{cases}$ for $2 \le i \le n$ and $h(x_i') = \begin{cases} 2i & if\ i\ is\ odd \\ 2i+1 & if\ i\ is\ even \end{cases}$ for $1 \le i \le n$

Then the number of edges labeled with 0 and 1 are $e_{h^*}(0) = e_{h^*}(1) = \frac{3n}{2}\ if\ n \equiv 2 (\text{mod } 4)$.

Also $e_{h^*}(0) = \left\lfloor \frac{3n}{2} \right\rfloor$ and $e_{h^*}(1) = \left\lceil \frac{3n}{2} \right\rceil$ if n is odd.

Here, $| e_{h^*}(0) - e_{h^*}(1) | \le 1$. Hence, $S'(K_{1,n})$ is a PQ- sum divisor cordial graph.

Theorem 2.12. Vertex switching of a cycle C_n admits PQ- sum divisor cordial labeling.

Proof. Let G be the cycle graph C_n and let G_x be the graph obtained from G by switching a vertex x of G. Let $x_i (1 \le i \le n)$ be the vertices of G. Without loss in generality we may assume that x = v_1. Define $h: V(G_{v_1}) \to \{1, 2, \dots, n\}$ by $h(x_i) = i$. Then the labeling h will induce the map $h^*: E(G) \to \{0, 1\}$. Also the number of edges labeled with 0 and 1 are

$e_{h^*}(0) = n - 2\ and\ e_{h^*}(1) = n - 3$.

Here, $| e_{h^*}(0) - e_{h^*}(1) | \le 1$. Hence, vertex switching of a cycle C_n is a PQ- sum divisor cordial graph.

Theorem 2.13. Vertex switching of a path graph P_n admits PQ- sum divisor cordial labeling.

Proof. Let G be a path graph P_n and let $x_i (1 \le i \le n)$ be the vertices of G. Let G_{x_i} be the graph obtained from G by switching a vertex x_i of G. Then we have $V(G) = V(G_{x_i})$.

Define $h: V(G_{x_i}) \to \{1, 2, \dots, n\}$ by $h(x_k) = \begin{cases} k+1 & if\ k < i \\ 1 & if\ k = i \\ k & if\ k > i \end{cases}$

Then the labeling h will induce the map $h^*: E(G) \to \{0, 1\}$. Also the number of edges labeled with 0 and 1 are $e_{h^*}(0) = e_{h^*}(1) = n - 3$.

Here, $| e_{h^*}(0) - e_{h^*}(1) | \le 1$. Hence, vertex switching of a path P_n is a PQ- sum divisor cordial graph.

References

[1] I. Cahit, Cordial graphs: A weaker version of graceful and harmonious graphs, Ars Combinatoria, Vol 23, PP.201-207, 1987.

[2] J.A. Gallian, A Dynamic Survey of Graph Labeling, The Electronic J.Combin., (2021), #DS6.

[3] F. Harary, Graph Theory, Addison-Wesley, Reading Mass, (1972).

[4] A. Lourdusamy and F. Patrick, Sum Divisor Cordial Graphs, Proyecciones Journal of Mathematics, 35(1), PP.119-136, (2016).

[5] A.Lourdusamy and F.Patrick, Sum Divisor Cordial labeling for star and ladder related graphs, Proyecciones Journal of Mathematics, 35(4), PP.437-455,(2016).

[6] R. Ponraj, S. Subbulakshmi and S. Somasundaram, PD- Prime Cordial Labeling of Graphs, Journal of Algorithms and Computation, 51 issue 2, December 2019, PP. 1-7.